Hall effects on convective hydromagnetic flow past a flat plate

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Hydromagnetic free convective flow past an infinite vertical, porous plate in the presence of a uniform transverse magnetic field has been considered taking Hall effects into account. Approximate solutions for the mean velocity, mean temperature and their related quantities are obtained. The influence of various dimensionless parameters is discussed

Keywords: *convection, Hall effects, mathematical models*

Recently considerable attention has been given to hydrodynamic and hydromagnetic boundary layer flows with or without Hall current effects. Convective flow of a viscous incompressible fluid past an infinite vertical plate was studied by Soundalgekar[1-4] and Gupta[5]. Pop and Soundalgekar[6] investigated the effects of Hall current on steady hydromagnetic boundary layer flow past a porous plate. This analysis was extended by Datta and Mazumder[7] who considered free convection effects in the vertical plate configuration. In this investigation the effects of Hall current on the unsteady hydromagnetic free convective flow past a vertical porous plate, when the free stream oscillates about a constant non-zero mean, is considered.

Basic equations of motion

The basic equations governing the physics of the problem are:

$$\nabla\times\boldsymbol{E}=0 \qquad \nabla\times\boldsymbol{J}=0$$

$$\boldsymbol{J}=\sigma\left[\boldsymbol{E}+\mu e\boldsymbol{q}\times\boldsymbol{H}-\frac{\mu_e}{n_e e}\boldsymbol{J}\times\boldsymbol{H}\right]$$

$$\frac{\partial \boldsymbol{q}}{\partial t}+(\boldsymbol{q}\cdot\nabla)\boldsymbol{q}=-\frac{1}{\rho}\nabla p+\frac{\eta}{\rho}\nabla^2\boldsymbol{q}+g\beta(T-T_\infty)+\frac{\mu_e}{\rho}\boldsymbol{J}\times\boldsymbol{H}$$

$$\frac{\partial \boldsymbol{T}}{\partial t}+(\boldsymbol{q}\cdot\nabla)T=\frac{k}{\rho C_p}\nabla^2 T+\frac{v}{2C_p}\left(\frac{\partial q_i}{\partial x_k}+\frac{\partial q_k}{\partial x_i}\right)^2+\boldsymbol{E}\cdot\boldsymbol{J} \qquad (1)$$

where the physical quantities have their usual meaning and assuming that the fluid is electrically quasi-neutral and ion slip and thermoelectric effects are negligible.

Since the plate is infinite, all physical quantities, except pressure, are functions of y' and t' only. The equation of continuity $\nabla\cdot\boldsymbol{q}=0$, gives $v'=-v_0(v_0>0)$, where $\boldsymbol{q}=(u',v',w')$. It is assumed that the induced magnetic field is negligible so that $\boldsymbol{H}=(0,H_0,0)$. This assumption is justified when the magnetic Reynold's number is very small[8]. The equation of conservation of electric charge $\nabla\cdot\boldsymbol{J}=0$ gives $j_{y'}=$constant, where $\boldsymbol{J}=(j_{x'},j_{y'},j_{z'})$. This constant is zero since $j_{y'}=0$ at the plate, which is electrically non-conducting. Thus $j_{y'}=0$ everywhere in the flow. We consider here the 'short circuit' case so $\boldsymbol{E}=\boldsymbol{O}$. Under these assumptions, the flow is now governed by:

$$\rho\left(\frac{\partial u'}{\partial t'}-v_0\frac{\partial u'}{\partial y'}\right)=-\frac{\partial p}{\partial x'}+\mu\frac{\partial^2 u'}{\partial y'^2}-\frac{\sigma\mu_e^2H_0^2}{1+m^2}(u'+mw')-\rho g \qquad (2)$$

$$O=-\frac{\partial p}{\partial y'} \qquad (3)$$

$$\left(\frac{\partial w'}{\partial t'}-v_0\frac{\partial w'}{\partial y'}\right)=-\frac{\partial p}{\partial z'}+\mu\frac{\partial^2 w'}{\partial z'^2}+\frac{\sigma\mu_e^2H_0^2}{1+m^2}(mu'-w') \qquad (4)$$

$$\frac{\partial T'}{\partial t'}-v_0\frac{\partial T'}{\partial y'}=\frac{k}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2}+\frac{v}{C_p}\left[\left(\frac{\partial u'}{\partial y'}\right)^2+\left(\frac{\partial w'}{\partial z'}\right)^2\right] \qquad (5)$$

The boundary conditions are:

$$u'=0 \qquad w'=0 \qquad T'=T_w' \qquad \text{at } y'=0$$
$$u'\to U'(t')=U_0(1+\varepsilon e^{i\omega' t'}) \qquad (6)$$
$$w'=0 \qquad T'\to T_\infty' \qquad \text{as } y'\to\infty$$

where ω' is the frequency of the fluctuating stream, U_0 the mean free stream velocity and εU_0 the amplitude of the free stream fluctuations. From Eqs (2) and (4), we have for the free-stream:

$$\rho\frac{\partial U'}{\partial t'}=-\frac{\partial p}{\partial x'}-\frac{\sigma\mu_e^2H_0^2U}{1+m^2}-\rho_\infty g \qquad (7)$$

$$0=-\frac{\partial p}{\partial x'}+\frac{\sigma\mu_e^2H_0^2mU}{1+m^2} \qquad (8)$$

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From Eqs (2)–(8), we obtain:

$$\rho\left(\frac{\partial u'}{\partial t'}-v_0\frac{\partial u'}{\partial y'}\right)=\rho\frac{\partial U'}{\partial t'}+\mu\frac{\partial^2 u'}{\partial y'^2}-\frac{\sigma\mu_e^2H_0^2}{1+m^2}[u'-v'+mw']+g(\rho_\infty-\rho) \tag{9}$$

$$\rho\left(\frac{\partial w'}{\partial t'}-v_0\frac{\partial w'}{\partial y'}\right)=\mu\frac{\partial^2 w'}{\partial y'^2}+\frac{\sigma\mu_e^2H_0^2}{1+m^2}[m(u'-v')-w'] \tag{10}$$

From the equation of state, we have:

$$g(\rho_\infty-\rho)=g\beta'\rho(T'-T'_\infty) \tag{11}$$

where β' is the coefficient of volume expansion.

Introducting non-dimensional quantities:

$$y=\frac{y'v_0}{v}\qquad t=\frac{t'v_0^2}{v}\qquad \omega=\frac{v\omega'}{v_0^2}$$

$$u=\frac{u'}{U_0}\qquad w=\frac{w'}{U_0}$$

$$U=\frac{U'}{U_0}\qquad \theta=\frac{T'-T'_\infty}{T'_w-T'_\infty}$$

$P=\dfrac{\rho v C_p}{k}$ (Prandtl number)

$G=\dfrac{vg\beta'(T'_w-T'_\infty)}{U_0v_0^2}$ (Grashof number)

$E=\dfrac{U_0}{C_p(T'_w-T'_\infty)}$ (Eckert number)

$M^2=\dfrac{\sigma\mu_e^2H_0^2v}{v_0^2}$ (magnetic parameter)

into Eqs (5), (9) and (10), the non-dimensional equations governing the motion are:

$$\frac{\partial u}{\partial t}-\frac{\partial u}{\partial y}=\frac{\partial U}{\partial t}+\frac{\partial^2 u}{\partial y^2}-\frac{M^2}{1+m^2}(u-U+mw)-G\theta \tag{12}$$

$$\frac{\partial w}{\partial t}-\frac{\partial w}{\partial y}=\frac{\partial^2 w}{\partial y^2}+\frac{M^2}{1+m^2}[m(u-U)-w] \tag{13}$$

$$\frac{\partial \theta}{\partial t}-\frac{\partial \theta}{\partial y}=\frac{1}{P}\frac{\partial^2 \theta}{\partial y^2}+E\left[\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial w}{\partial y}\right)^2\right] \tag{14}$$

The boundary conditions reduce to:

$$\begin{array}{llll} u=0 & w=0 & \theta=1 & \text{at } y=0 \\ u\to U(t) & w\to 0 & \theta\to 0 & \text{as } y\to\infty \end{array} \tag{15}$$

To solve these coupled non-linear equations, we assume that the unsteady flow is superimposed on the main steady flow. Hence, in the neighbourhood of the plate:

$$u=u_0+\varepsilon u_1 e^{i\omega t} \tag{16}$$

$$w=w_0+\varepsilon w_1 e^{i\omega t} \tag{17}$$

$$\theta=\theta_0+\varepsilon\theta_1 e^{i\omega t} \tag{18}$$

and for the free stream:

$$U(t)=1+\varepsilon e^{i\omega t} \tag{19}$$

where $\varepsilon \ll 1$. Substituting Eqs (16)–(19) in Eqs (12)–(14) and equating the like terms on both sides:

$$u_0''+u_0'-\frac{M^2}{1+m^2}[(u_0-1)+mw_0]=-G\theta_0 \tag{20}$$

$$w_0''+w_0'+\frac{M^2}{1+m^2}[m(u_0-1)-w_0]=0 \tag{21}$$

$$\theta_0''+P\theta_0'+PE[u_0'^2+w_0'^2]=0 \tag{22}$$

$$u_1''-\frac{M^2}{1+m^2}[(u_1-1)+mw_1]-i\omega(u_1-1)+u_1'=-G\theta_1 \tag{23}$$

$$w_1''+\frac{M^2}{1+m^2}[m(u_1-1)-w_1]-i\omega w_1+w_1'=0 \tag{24}$$

$$\theta_1''+2PE(u_0'u_1'+w_0'w_1')+P\theta_1'-i\omega P\theta_1=0 \tag{25}$$

where the prime denotes differentiation with respect to y. The corresponding boundary conditions reduce to:

$$\begin{array}{lllll} u_0=0 & w_0=0 & u_1=0 & w_1=0 & \theta_0=0 \\ \text{at } y=0 & & & & \\ u_0\to 1 & w_0\to 0 & u_1\to 0 & & \\ w_1\to 0 & \theta_0\to 0 & \theta_1\to 0 & \text{as } y\to\infty & \end{array} \tag{26}$$

Eqs (20)–(25) are still coupled and non-linear and hence difficult to solve. We expand the velocity and temperature in powers of E, the Eckert number, assuming that it is very small. This is justified in low speed incompressible flows. Hence we can write:

$$u_0(y)=u_{01}(y)+Eu_{02}(y)+O(E^2) \tag{27}$$

$$u_1(y)=u_{11}(y)+Eu_{12}(y)+O(E^2) \tag{28}$$

$$w_0(y)=w_{01}(y)+Ew_{02}(y)+O(E^2) \tag{29}$$

$$w_1(y)=w_{11}(y)+Ew_{12}(y)+O(E^2) \tag{30}$$

$$\theta_0(y)=\theta_{01}(y)+E\theta_{02}(y)+O(E^2) \tag{31}$$

$$\theta_1(y)=\theta_{11}(y)+E\theta_{12}(y)+O(E^2) \tag{32}$$

First, we proceed to obtain the solution for the mean flow and later on these results will be used to get the corresponding unsteady flow field, which will be presented in a future paper.

Substituting Eqs (27), (29) and (31) in Eqs (20) to (22), we get the following coupled differential equations for u_0, w_0 and θ_0:

$$u_{01}''+u_{01}'-\frac{M^2}{1+m^2}[u_{01}-1+mw_{01}]=-G\theta_{01} \tag{33}$$

$$w_{01}''+w_{01}'+\frac{M^2}{1+m^2}[m(u_{01}-1)-w_{01}]=0 \tag{34}$$

$$u_{02}''+u_{02}'-\frac{M^2}{1+m^2}[u_{02}+mw_{02}]=-G\theta_{02} \tag{35}$$

$$w_{02}''+w_{02}'+\frac{M^2}{1+m^2}[mu_{02}-w_{02}]=0 \tag{36}$$

$$\theta_{01}''+P\theta_{01}'=0$$

$$\theta_{02}''+P\theta_{02}'=-P[u_{01}'^2+w_{01}'^2] \tag{37}$$

The corresponding boundary conditions are:

$$\begin{array}{llll} u_{01}=0 & u_{02}=0 & w_{01}=0 & w_{02}=0 \\ \theta_{01}=1 & \theta_{02}=0 & \text{at } y=0 & \\ u_{01}\to 1 & u_{02}\to 0 & w_{01}\to 0 & \\ w_{02}\to 0 & \theta_{01}\to 0 & \theta_{02}\to 0 & \text{as } y\to\infty \end{array} \tag{38}$$

Solving Eqs (33)–(37) with these boundary conditions:

$$\begin{aligned} u_0(y)=u_{01}+Eu_{02}=&[1-e^{-\alpha_1 y}\{(1-A_1)\cos\beta_1 y \\ &+b_1\sin\beta_1 y\}-A_1 e^{-Py}]+EPG[C_1 e^{-Py} \\ &+C_2 e^{-(\alpha_1+P)y}\cos\beta_1 y+C_3 e^{-(\alpha_1+P)y}\sin\beta_1 y \\ &+C_4 e^{-2Py}+C_5 e^{-2\alpha_1 y}+C_6 e^{-\alpha_1 y}\cos\beta_1 y \\ &-C_7 e^{-\alpha_1 y}\sin\beta_1 y] \end{aligned} \tag{39}$$

$$\begin{aligned} w_0(y)=w_{01}+Ew_{02}=&[\{(1-A_1)\sin\beta_1 y \\ &-B_1\cos\beta^1 y\}e^{-\alpha_1 y}+B_1 e^{-Py}]+EPG[C_8 e^{-Py} \\ &+(C_9\cos\beta_1 y+C_{10}\sin\beta_1 y)e^{-(\alpha_1+P)y}-C_{11}e^{-2Py} \\ &-C_{12}e^{-2\alpha_1 y}-(C_7\cos\beta_1 y+C_6\sin\beta_1 y)e^{-\alpha_1 y}] \end{aligned} \tag{40}$$

$$\begin{aligned} \theta_0(y)=\theta_{01}+E\theta_{02}=&[e^{-Py}]+EP[a_5 e^{-Py}-a_1 e^{-2\alpha_1 y} \\ &-a_2 e^{-(\alpha_1+P)y}\cos\beta_1 y-a_3 e^{-(\alpha_1+P)y}\sin\beta_1 y-a_4 e^{-2Py}] \end{aligned} \tag{41}$$

where all the constants appearing in Eqs (39)–(41) are given in the appendix.

Knowing the mean velocity field and mean temperature field we can now calculate the mean skin-friction and mean rate of heat transfer from the plate. The mean skin friction in x- and z-directions are:

$$\begin{aligned} \tau_{mu}=&\alpha_1(1-A_1)-\beta_1 B_1+PA_1+EPG[-PC_1 \\ &-(\alpha_1+P)C_2+\beta_1 C_3-2PC_4-2\alpha_1 C_5-\alpha_1 C_6-\beta_1 C_7] \end{aligned}$$

and:

$$\begin{aligned} \tau_{mw}=&\alpha_1 B_1+\beta_1(1-A_1)-PB_1+EPG[-PC_8 \\ &-(\alpha_1+P)C_9-\beta_1 C_{10}+2PC_{11}+2\alpha_1 C_{12}+\alpha_1 C_7 \\ &-\beta_1 C_6] \end{aligned}$$

and the rate of heat transfer due to mean temperature q_m is:

$$q_m=-\left.\frac{d\theta_0}{dy}\right|_{y=0}$$

Discussion

We see from the solution that the steady state flow corresponding to $\varepsilon\to 0$ exhibits a boundary layer behaviour. Since the magnetic field is strong, the exponential e^{-Py} decays less rapidly than the other exponential terms and hence the thickness of the boundary layer is of order $1/P$ (assuming that P is less than, or of order, one). However, when $P\gg\alpha_1$ or order of α_1, $1/\alpha_1$ can be taken as a measure of the boundary layer thickness. In this case, the boundary layer thickness decreases with the increase in the magnetic parameter and increases with the increase in the Hall parameter. When the Grashof number G is small ($G\ll 1$), neglecting terms of order G in the solution, we have:

$$u_0\simeq 1-e^{-\alpha_1 y}\cos\beta_1 y \qquad w_0\simeq -e^{-\alpha_1 y}\sin\beta_1 y$$

which shows that the primary and the secondary velocity distribution are in the form of a logarithmic spiral similar to the Ekman velocity spiral for flow past a flat plate in a rotating fluid. Thus we may conclude that for small magnetic Reynold's numbers, Hall currents play a role similar to that of rotation.

The primary velocity $u_0(y)$ is plotted with y in Figs 1-3. Fig 1 shows that as P and m increase, the primary velocity decreases. From Fig 2 we observe that the velocity increases with increase in G whereas it decreases for negative values of G. Fig 3 shows that increases in magnetic parameter M give primary velocity decreases whereas greater viscous dissipation results in primary velocity increases.

In Figs 4–6 the secondary velocity $w_0(y)$ is plotted against y. Fig 4 shows that due to increase in P the velocity $w_0(y)$ decreases whereas it increases with increase in Hall

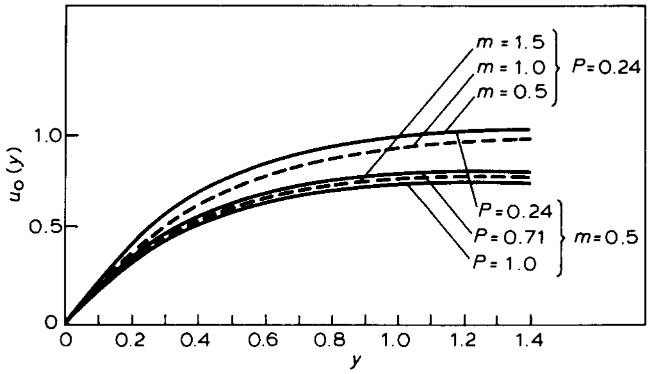

Fig 1 Mean velocity profile u_0 *for* $M^2=5$, $G=5$, $E=0.01$

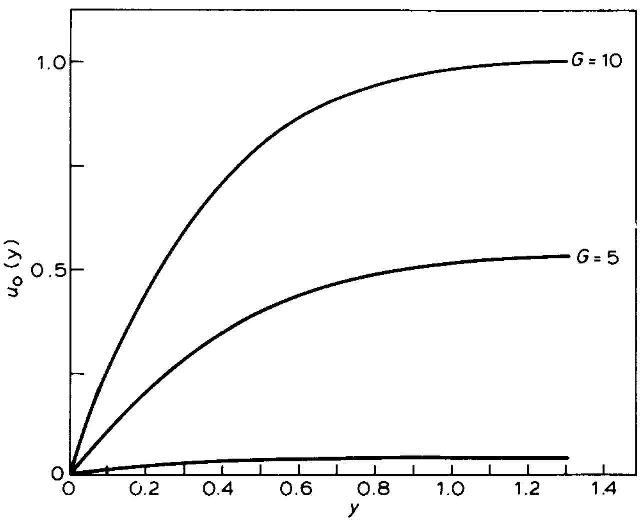

Fig 2 Mean velocity provile u_0 *for* $M^2=5$, $m=0.5$, $P=0.24$, $E=0.01$

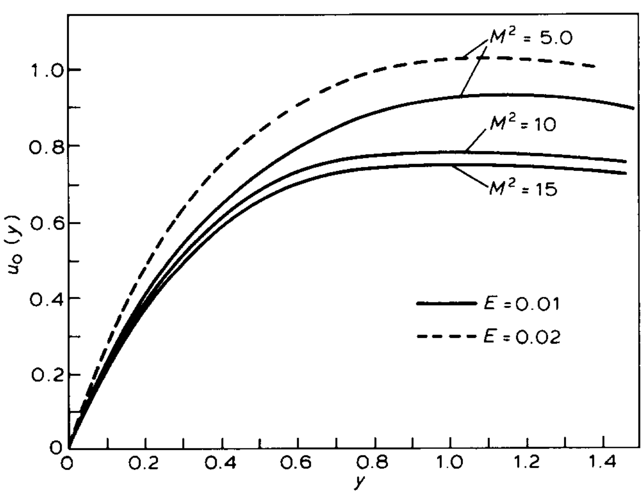

Fig 3 Mean velocity profile u_0 *for* $M^2=5$, $m=0.5$, $G=5$, $P=0.24$

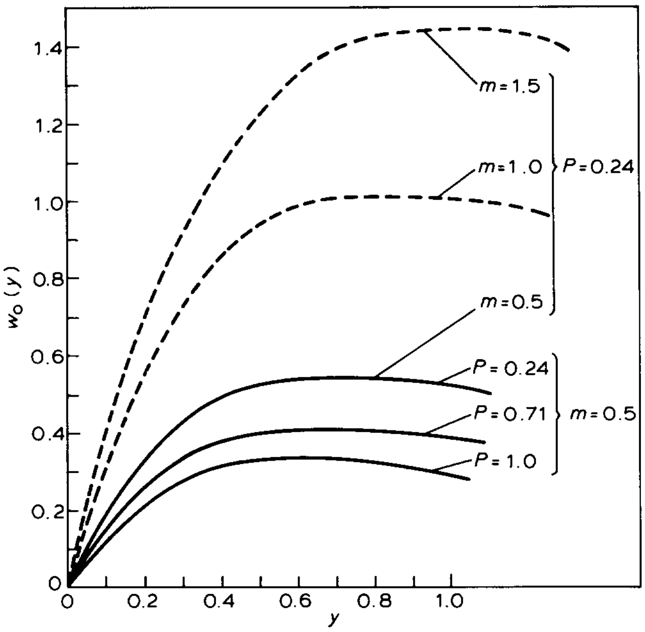

Fig 4 Mean velocity profile w_0 *for* $M^2=5$, $G=5.0$, $E=0.01$

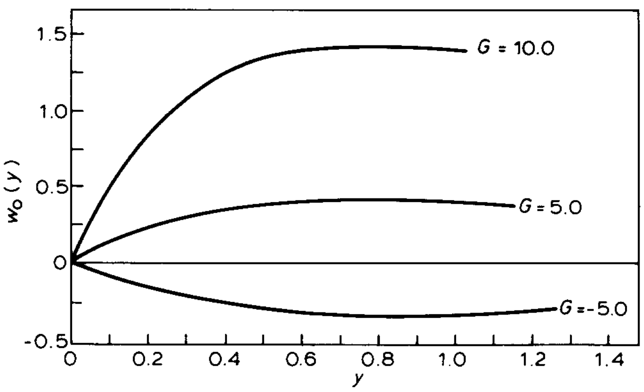

Fig 5 Mean velocity profile w_0 *for* $M^2=5$, $m=0.5$, $P=0.24$, $E=0.1$

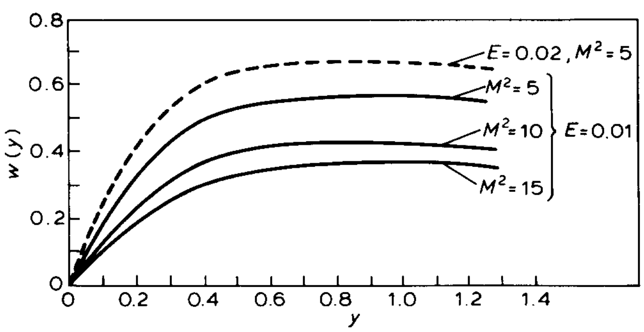

Fig 6 Mean velocity profile w_0 *for* $m=0.5$, $G=5$, $P=0.24$

parameter m. From Fig 5 we observe that the secondary velocity increases as G increases whereas for negative values of G there appears a reverse flow in the secondary velocity. Fig 6 shows that the secondary velocity decreases as magnetic parameter M increases, whereas it increases for greater heat dissipation.

The mean temperature distribution is shown in Figs 7 and 8 for various parameters. Figs 7 and 8 show that the mean temperature increases with increase in magnetic parameter M and Hall parameter m whereas it decreases with increase in E, G and P.

Numerical values of τ_{mu}, τ_{mw} and q_m are given in Tables 1 to 5 for $M^2=5.0$, $m=0.5$, $G=5.0$, $P=0.24$ and $E=0.01$.

From the tables we observe that the primary shear stress increases with increasing magnetic parameter M,

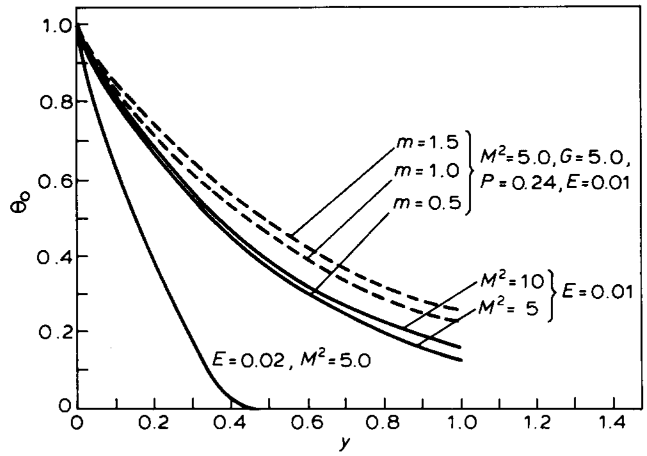

Fig 7 Mean temperature profile θ_0

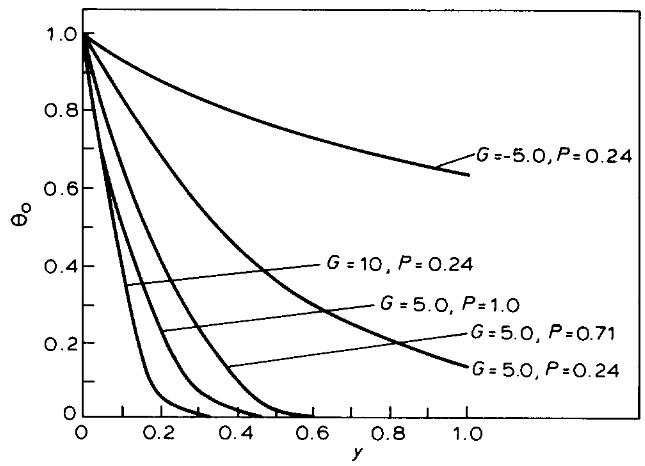

Fig 8 Mean temperature profile θ_0 *for* $M^2=5$, $E=0.01$, $m=0.5$

Table 1

M^2	τ_{mu}	τ_{mw}	q_m
5	0.7517	0.3013	0.2286
10	0.9025	0.3109	0.2308
15	1.0830	0.3432	0.3432

Table 2

m	τ_{mu}	τ_{mw}	q_m
0.5	0.7517	0.3013	0.2286
1.0	0.6439	0.4836	0.2278
1.5	0.5323	0.5840	0.2254

Table 3

G	τ_{mu}	τ_{mw}	q_m
5	0.7517	0.3013	0.2286
10	1.0540	0.7027	0.2153
−5	0.3048	−0.8632	0.2396

Table 4

P	τ_{mu}	τ_{mw}	q_m
0.24	0.7517	0.3013	0.2286
0.71	0.6897	0.2602	0.6856
1.0	0.6592	0.2502	0.9716

Table 5

E	τ_{mu}	τ_{mw}	q_m
0.01	0.7517	0.3013	0.2286
0.02	0.7713	0.3515	0.2172

Grashof number G and Eckert number E but decreases with increases in either Hall parameter m or Prandtl number P. The secondary shear stress increases with M, m, G and E but decreases with increasing Prandtl number P. It is also observed that the rate of heat transfer increases with increase in magnetic parameter M, Prandtl number P and decreases as parameter m, Grashof number G and Eckert number E increase.

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Appendix

$$\alpha = M^2/(1+m^2)$$
$$A = P^2 - P - \alpha$$
$$B = m\alpha$$
$$A_1 = GA/(A^2+B^2)$$
$$B_1 = GB/(A^2+B^2)$$
$$\alpha_1 = \frac{1}{2} + \frac{1}{2\sqrt{2}}[\{(1+4\alpha)^2+16B^2\}^{1/2}+(1+4\alpha)]^{1/2}$$
$$\beta_1 = \frac{1}{2\sqrt{2}}[\{(1+4\alpha)^2+16B^2\}^{1/2}-(1+4\alpha)]^{1/2}$$

$$R_3 = [A_1(1-A_1)-B_1^2](\alpha_1+P)+\beta_1 B$$
$$R_4 = [\alpha_1+P]B_1 - \beta[A_1(1-A_1)-B_1^2]$$
$$a_1 = (\alpha_1^2+\beta_1^2)[(1-A_1)^2+B_1^2]/[4\alpha_1^2-2\alpha_1 P]$$
$$a_2 = 2PR_3/[(\alpha_1+P)^2+\beta_1^2$$
$$a_3 = 2PR_4/[(\alpha_1+P)^2+\beta_1^2]$$
$$a_4 = \tfrac{1}{2}[A_1^2+B_1^2]$$
$$a_5 = a_1+a_2+a_4$$
$$P_1 = 4\alpha_1^2-2\alpha_1-\alpha$$
$$P_2 = (\alpha_1+P)^2-(\alpha_1+P)-\alpha-\beta_1^2$$
$$P_3 = 2\beta_1(\alpha_1+P)-\beta_1-E$$
$$P_4 = \tfrac{1}{2}(a_2P_2+a_3P_3)/(P_2^2+P_3^2)$$
$$P_5 = \tfrac{1}{2}(a_2P_3-a_3P_2)/(P_2^2+P_3^2)$$
$$P_6 = P_2$$
$$P_7 = 2\beta_1(\alpha_1+P)-\beta_1+\beta$$
$$P_8 - \tfrac{1}{2}(a_2P_6+a_3P_7)/(P_6^2+P_7^2)$$
$$P_9 = \tfrac{1}{2}[a_3P_6-a_2P_7]/(P_6^2+P_7^2)$$
$$P_{10} = 4P^2-2P-\alpha$$
$$P_{11} = \frac{Aa_5}{A^2+B^2} - \frac{a_1P_1}{P_1^2+B^2} - \frac{a_4P_{10}}{P_{10}^2+B^2} - P_4 - P_8$$
$$P_{12} = \frac{Ba_5}{A^2+B^2} - \frac{a_1B}{P_1^2+B^2} - \frac{a_4B}{P_{10}^2+B^2} + P_5 + P_9$$
$$C_1 = -a_5A/(A^2+B^2)$$
$$C_2 = P_4+P_8$$
$$C_3 = P_9-P_5$$
$$C_4 = a_4P_{10}/(P_{10}^2+B^2)$$
$$C_5 = a_1P_1/(P_1^2+B^2)$$
$$C_6 = P_{11}$$
$$C_7 = P_{12}$$
$$C_8 = a_5B/(A^2+B^2)$$
$$C_9 = P_5+P_9$$
$$C_{10} = P_4-P_8$$
$$C_{11} = a_4B/(P_{10}^2+B^2)$$
$$C_{12} = a_1B/(P_1^2+B^2)$$